Correction DM2 du 1 octobre.

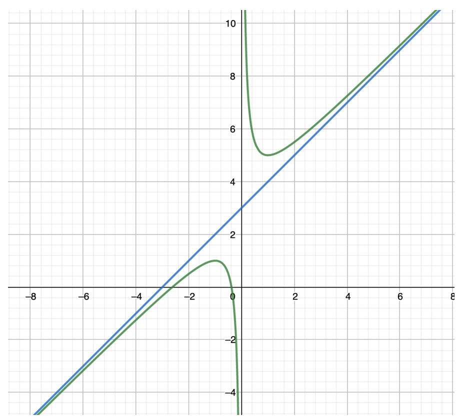
Ex 56 page 98

1.
$$\left.\begin{array}{r}\lim\limits_{x\to 0} x^2+3x+1=1\\ \lim\limits_{x\to 0^-} x=0^-\end{array}\right\}\Rightarrow \lim_{x\to 0^-} f(x)=-\infty \qquad \text{et} \qquad \left.\begin{array}{r}\lim\limits_{x\to 0} x^2+3x+1=1\\ \lim\limits_{x\to 0^+} x=0^-\end{array}\right\}\Rightarrow \lim_{x\to 0^+} f(x)=+\infty$$

2.
$$f(x)=\frac{x^2\left(1+\frac{3}{x}+\frac{1}{x^2}\right)}{x}=x\left(1+\frac{3}{x}+\frac{1}{x^2}\right) \qquad \text{donc} \qquad \left.\begin{array}{r}\lim\limits_{x\to+\infty} =1+\frac{3}{x}+\frac{1}{x^2}=1\\ \lim\limits_{x\to+\infty} x=+\infty\end{array}\right\}\Rightarrow \lim_{x\to+\infty} f(x)=+\infty$$

$$\text{et} \qquad \left.\begin{array}{r}\lim\limits_{x\to-\infty} =1+\frac{3}{x}+\frac{1}{x^2}=1\\ \lim\limits_{x\to-\infty} x=-\infty\end{array}\right\}\Rightarrow \lim_{x\to+\infty} f(x)=-\infty$$

3. .



4.
$$g(x)=f(x)-(x+3)=\frac{x^2+3x+1-x(x+3)}{x}=\frac{1}{x} \qquad \text{donc} \qquad \lim_{x\to+\infty} g(x)=0 \qquad \text{et} \qquad \lim_{x\to-\infty} g(x)=0$$

Donc la droite d'équation $y=x+3$ est une asymptote oblique à la représentation graphique de f.

Ex 85 page

1.
$$f(x)=\frac{x^3\left(1-\frac{1}{x}+\frac{3}{x^3}\right)}{x^2\left(1-\frac{1}{x^2}\right)}=x\frac{\left(1-\frac{1}{x}+\frac{3}{x^3}\right)}{\left(1-\frac{1}{x^2}\right)}$$

$$\left.\begin{array}{r}\left.\begin{array}{r}\lim\limits_{x\to+\infty} 1-\frac{1}{x}+\frac{3}{x^3}=1\\ \lim\limits_{x\to+\infty} 1-\frac{1}{x^2}=1\end{array}\right\}\Rightarrow \lim\limits_{x\to+\infty}\frac{\left(1-\frac{1}{x}+\frac{3}{x^3}\right)}{\left(1-\frac{1}{x^2}\right)}=1\\ \lim\limits_{x\to+\infty} x=+\infty\end{array}\right\}\lim_{x\to+\infty} f(x)=\lim_{x\to+\infty} x\frac{\left(1-\frac{1}{x}+\frac{3}{x^3}\right)}{\left(1-\frac{1}{x^2}\right)}=+\infty$$

$$\left.\begin{array}{r}\left.\begin{array}{r}\lim\limits_{x\to-\infty} 1-\frac{1}{x}+\frac{3}{x^3}=1\\ \lim\limits_{x\to-\infty} 1-\frac{1}{x^2}=1\end{array}\right\}\Rightarrow \lim\limits_{x\to-\infty}\frac{\left(1-\frac{1}{x}+\frac{3}{x^3}\right)}{\left(1-\frac{1}{x^2}\right)}=1\\ \lim\limits_{x\to-\infty} x=-\infty\end{array}\right\}\lim_{x\to-\infty} f(x)=\lim_{x\to-\infty} x\frac{\left(1-\frac{1}{x}+\frac{3}{x^3}\right)}{\left(1-\frac{1}{x^2}\right)}=-\infty$$

$$\left.\begin{array}{r}\lim\limits_{x\to-1} x^3-x^2+3=3\\ \lim\limits_{x\to-1^-} x^2-1=0^+\end{array}\right\}\Rightarrow \lim_{x\to-1^-} f(x)=+\infty \qquad \text{et} \qquad \left.\begin{array}{r}\lim\limits_{x\to-1} x^3-x^2+3=3\\ \lim\limits_{x\to-1^+} x^2-1=0^-\end{array}\right\}\Rightarrow \lim_{x\to-1^+} f(x)=-\infty$$

$$\left.\begin{array}{r}\lim\limits_{x\to 1} x^3-x^2+3=3\\ \lim\limits_{x\to 1^-} x^2-1=0^-\end{array}\right\}\Rightarrow \lim_{x\to -1^-} f(x)=-\infty \qquad \text{et} \qquad \left.\begin{array}{r}\lim\limits_{x\to 1} x^3-x^2+3=3\\ \lim\limits_{x\to 1^+} x^2-1=0^+\end{array}\right\}\Rightarrow \lim_{x\to -1^+} f(x)=+\infty$$

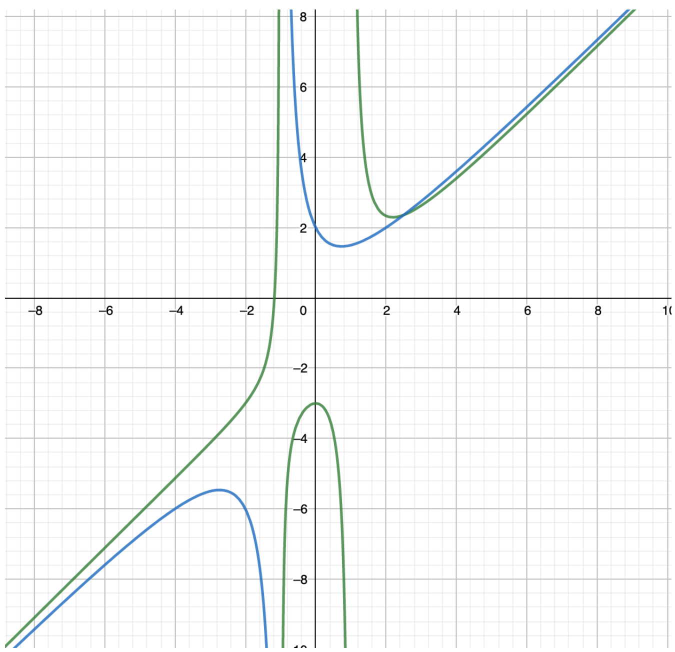
$$g(x)=\frac{x^2\left(1+\frac{2}{x^2}\right)}{x\left(1+\frac{1}{x}\right)}=x\frac{\left(1+\frac{2}{x^2}\right)}{\left(1+\frac{1}{x}\right)}$$

$$\left.\begin{array}{r}\left.\begin{array}{r}\lim\limits_{x\to+\infty} 1+\frac{2}{x^2}=1\\ \lim\limits_{x\to+\infty} 1+\frac{1}{x}=1\end{array}\right\}\Rightarrow \lim\limits_{x\to+\infty}\frac{\left(1+\frac{2}{x^2}\right)}{\left(1+\frac{1}{x}\right)}=1\\ \lim\limits_{x\to+\infty} x=+\infty\end{array}\right\}\lim_{x\to+\infty} g(x)=+\infty$$

$$\left.\begin{array}{r}\left.\begin{array}{r}\lim\limits_{x\to-\infty} 1+\frac{2}{x^2}=1\\ \lim\limits_{x\to-\infty} 1+\frac{1}{x}=1\end{array}\right\}\Rightarrow \lim\limits_{x\to-\infty}\frac{\left(1+\frac{2}{x^2}\right)}{\left(1+\frac{1}{x}\right)}=1\\ \lim\limits_{x\to-\infty} x=-\infty\end{array}\right\}\lim_{x\to-\infty} g(x)=-\infty$$

$$\left.\begin{array}{r}\lim\limits_{x\to -1} x^2+2=3\\ \lim\limits_{x\to -1^-} x+1=0^-\end{array}\right\}\Rightarrow \lim_{x\to -1^-} g(x)=-\infty \qquad \text{et} \qquad \left.\begin{array}{r}\lim\limits_{x\to -1} x^2+2=3\\ \lim\limits_{x\to -1^+} x+1=0^+\end{array}\right\}\Rightarrow \lim_{x\to -1^+} g(x)=+\infty$$

2. .



3. (a)

$$f(x)-g(x)=\frac{x^3-x^2+3}{(x-1)(x+1)}-\frac{(x^2+2)(x-1)}{(x+1)(x-1)}=\frac{x^3-x^2+3-(x^3-x^2+2x-2)}{x^2-1}=\frac{-2x+5}{x^2-1}$$

(b) On obtient donc le tableau de signe :

x	$-\infty$		-1		1		$\frac{5}{2}$		$+\infty$
$-2x+5$		$+$		$+$		$+$	0	$-$	
x^2-1		$+$	0	$-$	0	$+$		$+$	
$f(x)-g(x)$		$+$	‖	$-$	‖	$+$	0	$-$	
position		$\dfrac{\mathscr{C}_f}{\mathscr{C}_g}$	‖	$\dfrac{\mathscr{C}_g}{\mathscr{C}_f}$	‖	$\dfrac{\mathscr{C}_f}{\mathscr{C}_g}$		$\dfrac{\mathscr{C}_g}{\mathscr{C}_f}$	

(c)

$$f(x) - g(x) = \frac{1}{x} \times \frac{-2 + \frac{5}{x}}{1 - \frac{1}{x^2}}$$

$$\left.\begin{array}{r} \left.\begin{array}{r} \lim\limits_{x \to +\infty} -2 + \frac{5}{x} = -2 \\ \lim\limits_{x \to +\infty} 1 - \frac{1}{x^2} = 1 \end{array}\right\} \Rightarrow \lim\limits_{x \to +\infty} \frac{-2 + \frac{5}{x}}{1 - \frac{1}{x^2}} = -2 \\ \lim\limits_{x \to +\infty} x = +\infty \end{array}\right\} \lim_{x \to +\infty} f(x) - g(x) = \lim_{x \to +\infty} \frac{1}{x} \times \frac{-2 + \frac{5}{x}}{1 - \frac{1}{x^2}} = 0$$

$$\left.\begin{array}{r} \left.\begin{array}{r} \lim\limits_{x \to -\infty} -2 + \frac{5}{x} = -2 \\ \lim\limits_{x \to -\infty} 1 - \frac{1}{x^2} = 1 \end{array}\right\} \Rightarrow \lim\limits_{x \to -\infty} \frac{-2 + \frac{5}{x}}{1 - \frac{1}{x^2}} = -2 \\ \lim\limits_{x \to -\infty} x = -\infty \end{array}\right\} \lim_{x \to -\infty} f(x) - g(x) = \lim_{x \to -\infty} \frac{1}{x} \times \frac{-2 + \frac{5}{x}}{1 - \frac{1}{x^2}} = 0$$

On peut donc remarquer que les courbes de f et g *se rapproche* en l'infini.